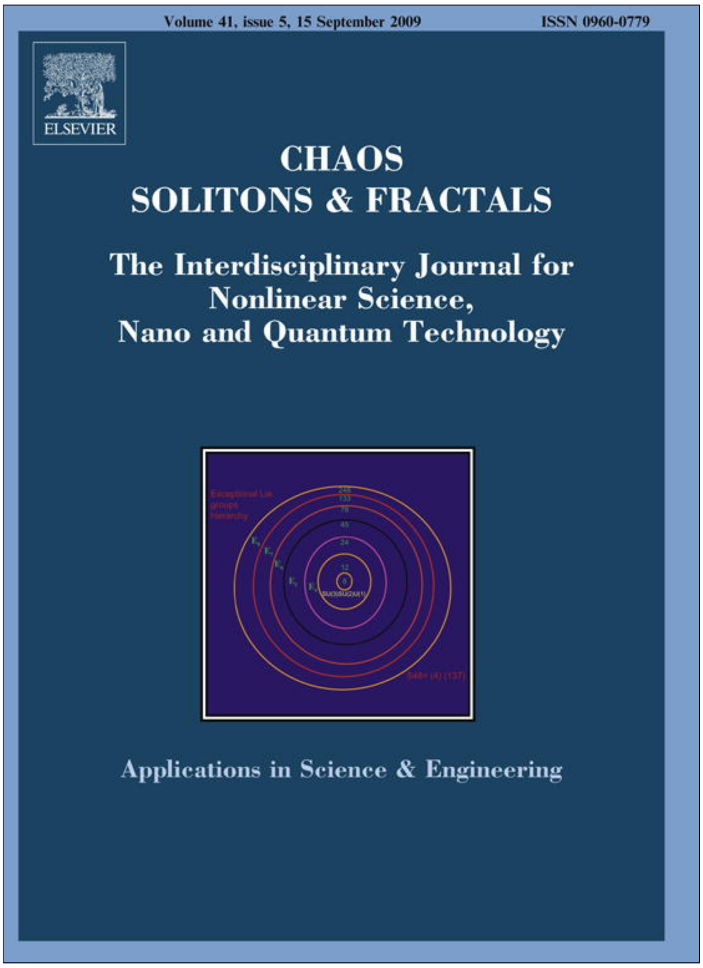
Volume 41, issue 5, 15 September 2009
ISSN 0960-0779
ELSEVIER
CHAOS
SOLITONS & FRACTALS
The Interdisciplinary Journal for
Nonlinear Science,
Nano and Quantum Technology
Applications in Science & Engineering

Chaos, Solitons and Fractals 41 (2009) 2474–2483

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

journal homepage: www.elsevier.com/locate/chaos

Fractal analysis of time series in oil and gas production

Arif A. Suleymanov [a,*], Askar A. Abbasov [b], Aydin J. Ismaylov [c]

[a] Azerbaijan State Oil Academy, Reservoir and Petroleum Engineering Department, Azadlig Avenue 20, AZ1010, Baku, Azerbaijan
[b] Salyan Oil Ltd., ISR Plaza Business Center, Nizami Street, 340, AZ1000, Baku, Azerbaijan
[c] Caspian Energy Group, Salamzadeh Street, 65B, AZ1069, Baku, Azerbaijan

ARTICLE INFO

Article history:
Accepted 16 September 2008

Communicated by T. Kapitaniak

ABSTRACT

The peculiarities of fractal characteristics' calculations for time series are described in this article. An algorithm for calculation of fractal dimension is suggested. It has been proved that the suggested method possesses high accuracy and the rapidity of convergence on the limited number of measurements compared to the methods of covering.

The criteria of early diagnosis for changes in the condition of hydrodynamic processes, which do not vary by fractal dimension, have been recommended.

The presented method is applicable for practical engineering calculations with self-affine, chaotic data, usually with relatively limited number of measurements. It is quite a simple method for calculation of fractal dimension, algorithm can be easily realized and it should be useful for engineers.

The applicability of the proposed algorithm for fractal dimension calculation and early diagnosis criteria of qualitative changes in the behaviour of various dynamic systems has been tested both on simulated as well as practical examples of oil and gas production.

1. Introduction

The concept of fractals [1,2] at present times has gained a wide application in numerous spheres of natural science. Thus, the abilities of fractal geometry found their application in studies of different processes of oil and gas production [3–6].

Fractal analysis is used to interpret the results of geophysical and hydrodynamic measurements (seismic measurements, logging, fluid samples, coring, pressure transient tests etc.) [7].

There is a number of works about experimental identification of fractal dimension of porous media. The spatial fractal dimension of a real rock sample was defined using SEM (Scanning Electron Microscope) [14]. A method for definition of fractal dimension from experimental isotherm of gas adsorption has been suggested [8]. Alternative ways to define fractal characteristics' of a reservoir using PBU (Pressure Build-Up) curves and reaction of the reservoir to pressure changes is discussed in works [9]. There is a number of studies devoted to filtration in fractal-heterogeneous medias [10,11], including the relative permeability curves [12].

Fractal approach is also used when modelling filtration in a porous media. The equations of movement and methods of PBU analysis in the reservoirs with fractal structure have been presented [6,13].

The analysis of experimental and field information shows that the fluctuations which emerge in technical systems very often have deterministic character. They are created by the system itself. For this reason they may serve as important information source about its inner characteristics.

One of the directions of using fractal characteristic in technical diagnostics is tied to the fact that the time series of the main well performance indicators measurements quite often have fractal structure.

* Corresponding author.
E-mail address: petrotech@azeuro.net (A.A. Suleymanov).

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doi:10.1016/j.chaos.2008.09.039

In work [6], a possibility to apply correlation dimension and wavelet analysis has been shown. The correlation dimension a wavelet analysis are used for definition of the drilling bit wear degree in order to change it out in timely manner. The pressure fluctuations of the drilling mud and axial weight on bit have been used for calculations.

The fractal analysis is also used for definition of optimal gas lift regimes. Hurst exponent, which is calculated using rate fluctuations of an oil well, is used for this [6].

The modern technologies enable data acquisition of oil well performance indicators (rate, pressure, temperature etc.) in real time and to use it to analyse the current reservoir conditions.

Hence it is very genuine to use simple and reliable methods of time series fractal dimension definition for engineering calculations.

2. The peculiarities of fractal dimension of time series

The concept of fractal is not applied solely to topological objects. Time series follow the laws of fractal geometry. The scale invariance for time series emerges when during the same period of time T the process is measured by different steps Δt_n and new points are added to the curve.

Fractal laws can be observed in the behaviour of such time processes of oil and gas production as fluctuations of production, pressure and etc. This happens when the time step is reduced and at the same time more and more peculiarities of studied parameters are revealed. The character of their fluctuations depends both on external influences as well as on unsteady state process of multiphase systems' filtration. It also carries the information about the state and behaviour of the formation [14]. The definition of fractal characteristics of production well technological parameters allows conducting early diagnosis of changes in system "formation-well" and regulating the well regime in a timely manner. The aim is to optimize the process of field development. It has been shown in this study [15] that the fractal dimensions of fluid and pressure fluctuations in a well change as the sweeping water front approaches closer. This allows predicting water cut.

However, a reliable calculation of fractal characteristics D for time series, in connection with finiteness of measurement step, interfaces with the following complications: necessity of long-term studies to acquire large amount of measurements and the changing behaviour of dynamic process during the measurement time [16]. In order to tie together local changeability of a dynamic process with fractal dimension of time series it is necessary to define D locally. It is therefore vital that the method of fractal dimension calculation for time processes possesses rapidity of convergence in the limited number of measurements.

2.1. Determination of fractal dimension of time series

At present, the fractal dimension of time series is traditionally determined in the same way as a curve of some factor is built for continuous curves according to the measurements with a set time step of Δt during the measurement interval of O. The length of the built curve is determined by methods of covering, when the estimated length L, equals δ – the length of the rulers (boxes), multiplied the number of such rulers (boxes) needed to cover the measured curve. The dimension D can be determined using dependence $L \sim \delta^{1-D}$ in logarithmic scales.

However, in case the method of covering suits the continuous curves, then this method is very coarse for time dependencies. This is due to the fact that measurements of technological parameters are discrete. The discrete points get connected by series of lines when building a graph for visualization. The measurements themselves, not the lines connecting them on the graph, should be taken into account when calculating fractal dimension for time series [17].

This study suggests a method for fractal dimension calculation of time series which possesses as rapid convergence on limited amount of measurements as well as simplicity of realization.

The suggested algorithm for fractal dimension calculation consists of the following: Let during a time interval T, the n amount of measurements have been carried out of y_i dynamic process with time step of Δt.

Let's put together the sequence of selected measurements out of all measurements according to the following rule:

The first selection contains all measurements of time interval T

$$y_1, y_2, y_3, \ldots, y_{n-1}, y_n \quad (\text{Table 1, column } A_1);$$

The second selection consists of measurements, which stand away from each other at the distance of $2^*\Delta t$

$$y_1, y_3, y_5, \ldots, y_{n-2}, y_n \quad (\text{Table 1, column } A_2);$$

The third selection consists of measurements, which are stand away from each other at the distance of $3^*\Delta t$

$$y_1, y_4, y_7, \ldots, y_{n-3}, y_n \quad (\text{Table 1, column } A_3)$$

etc.

The length of time dependencies L for selection we determine as the sum of absolute values of differences between neighbouring measurements of the investigated parameter for the given selection.

Thus, the length of the first selection will be equal to (the sum of the values in column B_1 of Table 1)

$$|y_1 - y_2| + |y_2 - y_3| + \ldots + |y_{n-1} - y_n|. \tag{1}$$

Table 1
Data for calculation

A_1	A_1	A_2	B_2	A_3	B_3	...	A_m	B_m
y_1		y_1		y_1		...	y_1	
y_2	$\|y_1-y_2\|$		$\|y_1-y_3\|$		$\|y_1-y_4\|$	...		
y_3	$\|y_2-y_3\|$	y_3				...	...	$\|y_1-y_{1+m}\|$
y_4	$\|y_3-y_4\|$		$\|y_3-y_5\|$	y_4		...		
y_5	$\|y_4-y_5\|$	y_5			$\|y_4-y_7\|$	...	y_{1+m}	
y_6	$\|y_5-y_6\|$		$\|y_5-y_7\|$			...		
y_7	$\|y_6-y_7\|$	y_7		y_7		...		
...	...	...	...	...	...	...	...	...
y_{n-2}	$\|y_{n-3}-y_{n-2}\|$	y_{n-2}				...		
y_{n-1}	$\|y_{n-2}-y_{n-1}\|$		$\|y_{n-2}-y_n\|$		$\|y_{n-3}-y_n\|$	...		$\|y_{n-m}-y_n\|$
y_n	$\|y_{n-1}-y_n\|$	y_n		y_n		...	y_n	

The length of the second selection will be equal to (the sum of the values in column B_2 of Table 1)

$$|y_1 - y_3| + |y_3 - y_5| + \ldots + |y_{n-2} - y_n|.$$

The length of the third selection is determined in the same way (the sum of the values in column B_3 of Table 1)

$$|y_1 - y_4| + |y_4 - y_7| + \ldots + |y_{n-3} - y_n|.$$

Hence, the length of a curve is determined in depending on the time step of measurements $\Delta t_m = m^*\Delta t$, and not the value of δ.

In consequence of limitation in variation of the measured parameters, the length of the curve L at small values of Δt_m is best described by the following dependence $L \sim \Delta t_m^{1-D}$.

Taking into account that Δt_m is in inverse proportion to $k = \frac{n-1}{m}$ (to the number of lengths which divide the time interval T), we can write down the following:

$$L \sim k^{D-1}. \tag{2}$$

Lets rebuild this dependence in $\log L - \log k$ coordinates. When the values of k are high, the given dependence lies onto a straight line. The slope of this straight line defines the value of D.

Experimental studies allow to confirm that the suggested method of fractal dimension calculation possesses high accuracy and rapidity of convergence compared to the methods of covering.

Moreover, the given approach allows to avoid the discrepancy units of measured parameters and time when determining the fractal dimension [18].

The applicability of the suggested algorithm is tested on a modelled example where the values of Weierstrass–Mandelbrot $W(t)$ [19,20] fractal dimension D are taken into account as the values of the studied process:

$$W(t) = \sum_{N=-\infty}^{\infty} \frac{\left(1 - e^{ib^N t}\right) e^{i\varphi_N}}{b^{(2-D)N}}, \tag{3}$$

where φ_N – arbitrary phase; parameter D varies in the diapason of $1 < D < 2$. When $\varphi_N = 0$ the Weierstrass–Mandelbrot $W(t)$ function takes a simpler form:

$$C(t) = \sum_{N=-\infty}^{\infty} \frac{(1 - \cos(b^N t))}{b^{(2-D)N}}. \tag{4}$$

The graph of $C(t)$ function at different values is presented on Fig. 1 (the data is normalized according to maximum and minimum values).

It is notable that when D values are small the function is almost smooth, and when the values increase it starts to fluctuate.

An example of fractal dimension calculation for the case of $D = 1.6$ and $n = 2500$ is shown on Fig. 2. The calculated value of fractal dimension according to the suggested method made up 1.597.

In [17], the suggested method was compared with the method of covering. The fractal dimension was calculated according to Weierstrass–Mandelbrot fractal function with given D both with the suggested as well as with the covering methods. The comparison showed that the covering method needs significantly larger number of measurements in order the calculated value of D be close to the given one.

2.2. Sensitivity of calculating algorithm according to the number of data

Cases when n= 100; 250; 500; 1000; 1500; 2000; 2500 were looked at when aiming to investigate the influence of measurements' quantity onto the preciseness of fractal dimension determination.

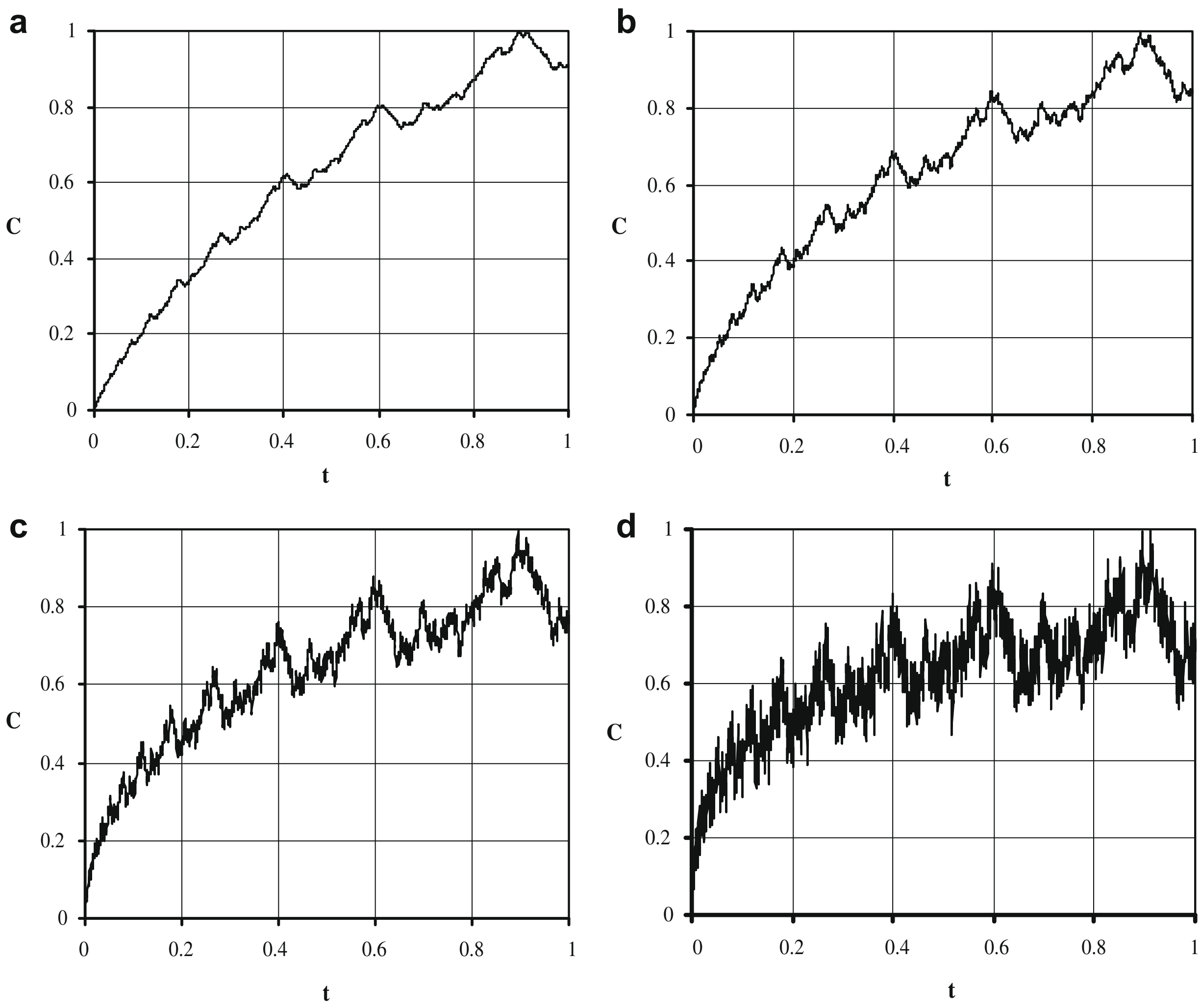

Fig. 1. Normalized function $C(t)$ for $n = 2500$ at different values of fractal dimension D: (a) $D = 1.2$; (b) $D = 1.4$; (c) $D = 1.6$; (d) $D = 1.8$.

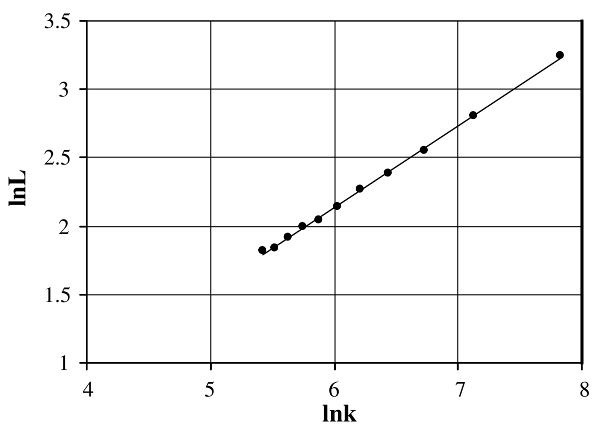

Fig. 2. Dependence $\ln L = f(\ln k)$ for $n = 2500$, $b = 1.5$ at $D = 1.6$.

The results of the carried out calculations are presented in Table 2.

From Table 2 it is clear that there is a good correspondence between the set and calculated values of D. At that, the accuracy is higher when the quantity of measurement points increases.

When comparing to the method of covering, the suggested in the study method, together with simplicity, possesses high accuracy and rapidity of convergence at limited amount of measurements.

Table 2
Comparison of the given and rated values of fractal dimension

No of measurements n	Fractal dimension D			
	$D = 1.2$	$D = 1.4$	$D = 1.6$	$D = 1.8$
100	1.13	1.30	1.50	1.68
250	1.16	1.36	1.60	1.74
500	1.18	1.38	1.62	1.88
1000	1.17	1.37	1.58	1.77
1500	1.17	1.38	1.59	1.80
2000	1.17	1.38	1.59	1.79
2500	1.18	1.40	1.60	1.80

2.3. Accuracy of calculating algorithm

The accuracy of fractal dimension calculation can be enhanced if all existing measurements are taken into account.

Thus, for $\Delta t_2 = 2^*\Delta t$ depending on definition of initial point, 2 values for length $L_2 : L_2^1$ and L_2^2 could be calculated, and 2 values $k_2 : k_2^1$ and k_2^2 correspondingly, where

$$L_2^1 = |y_1 - y_3| + |y_3 - y_5| + |y_7 - y_5| + \dots,$$
$$L_2^2 = |y_2 - y_4| + |y_6 - y_4| + |y_8 - y_6| + \dots.$$

For $\Delta t_3 = 3^*\Delta t$ depending on selection of initial point, 3 values $L_3 : L_3^1,\ L_3^2,\ L_3^3$ can be calculated and corresponding to them 3 values $k_3 : k_3^1,\ k_3^2,\ k_3^3$ and so on.

In this case fractal dimension D can be identified by arithmetical mean values L_m and k_m from the following dependence:

$$L_{\text{average}} \sim k_{\text{average}}^{D-1}, \tag{5}$$

where $L_{\text{average}} = \frac{L_m}{m} = \frac{L_m^1 + L_m^2 + \dots + L_m^m}{m}$, $k_{\text{average}} = \frac{k_m}{m} = \frac{k_m^1 + k_m^2 + \dots + k_m^m}{m}$.

Comparing Fig. 3 and Fig. 4 (function $C(t)$ at $D = 1.4$, $b = 1.5$ and $n = 250$) it is seen that the dependence $\log L - \log k$ is significantly better straightened when the length of the L and k curves is averaged (when all the existing measurements are taken into account).

3. Analysis of processes with close fractal dimensions

Fractal dimension and Huarst exponent are used as integrated factors when solving practical tasks. They characterize the peculiarities of the investigated object. This allows diagnosing the changes in conditions of technological processes.

The deficiency of the fractal analysis, in its classical statement, is the fact that numerous technological processes, which are characterized by factors' fluctuations (in particular a wide range of stationary random or close to them processes) do not divide on fractal coordinate space [20].

In order to increase the effectiveness of process condition diagnosis, a method, shown below, is suggested.

In case, when time series $y(t)$ and $z(t)$ are compared, the difference between their fractal dimensions is small. Their distinction can be estimated either according to changes in coefficient A from the following dependence:

$$L = Ak^{D-1}, \tag{6}$$

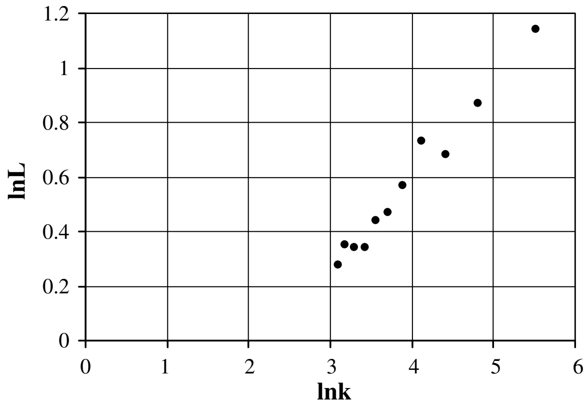

Fig. 3. Dependence $\ln L = f(\ln k)$ for $n = 250$, $b = 1.5$ at $D = 1.4$.

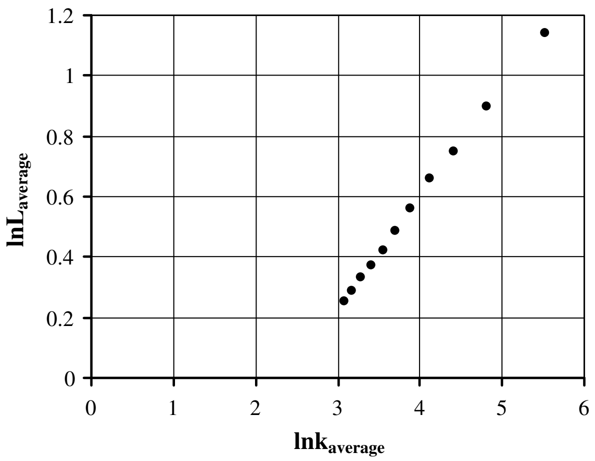

Fig. 4. Dependence $\ln L_{\text{average}} = f(\ln k_{\text{average}})$ for $n = 250$, $b = 1.5$ at $D = 1.4$.

or, in case initial data does not straighten in logarithmic coordinates $\log L - \log k$, using the method of specific length comparison:

$$l = \frac{L_1}{k_1}, \tag{7}$$

where L_1 we find from Eq. (1) and $k_1 = n - 1$, or comparing values

$$\sqrt{\frac{\sum_{i=2}^{n}(y_{i-1} - y_i)^2}{n-1}} = \sqrt{\frac{(y_1 - y_2)^2 + (y_2 - y_3)^2 + \ldots + (y_{n-1} - y_n)^2}{n-1}}. \tag{8}$$

At that, initial time series $y(t)$ and $z(t)$ are normalized, for example, each of them according to their maximum and minimum values:

$$Y_i = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}} \text{ and } Z_i = \frac{z_i - z_{\min}}{z_{\max} - z_{\min}}. \tag{9}$$

Such transformation (normalization) of initial time series is not reflected on the value of their fractal dimensions and, at the same time, allows to track the change in local fractal dimension of the same process at different stages of its development and to compare different characteristics of the process, which were measured at the same period of time.

Sensitivity of coefficient A to non-significant changes of the investigated process, which are difficult to detect by the value of fractal dimension, can be seen from the following example:

Lets regard the normalized values of function $C(t)$ at the same value $D = 1.4$, $n = 1000$ and varying b: 1.5 and 1.2 (Fig. 5).

As seen from Fig. 6, the initial data is well straightened in logarithmic coordinates and the values of fractal dimension, which are defined from slope of straight line, do not differ: 1.37 and 1.37.

At the same time, coefficients A for the given curves, after expansion from logarithm, differ significantly and equal to 0.398 and 0.284 correspondingly. The difference of these curves can be seen according to the values of specific lengths, which are equal to 0.005 and 0.004 correspondingly.

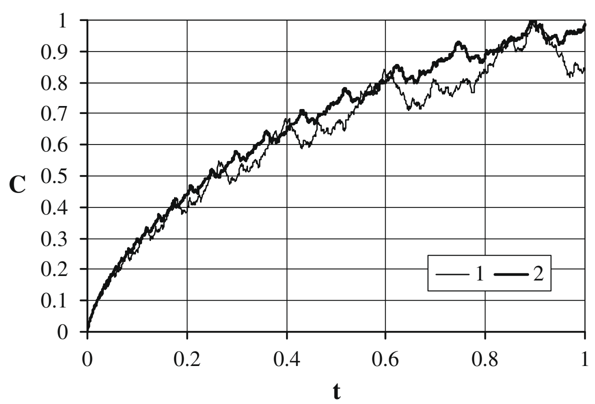

Fig. 5. Graphs of normalized values of function $C(t)$ at different b: 1 – at $b = 1.2$; 2 – at $b = 1.5$.

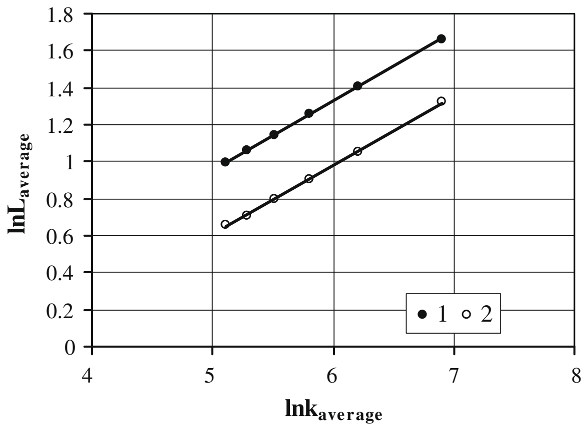

Fig. 6. Dependence $\ln L_{\text{average}} = f(\ln k_{\text{average}})$ at different b: 1– at $b = 1.5$; 2 – at $b = 1.2$.

A similar analysis has been carried out for $n = 1000$ random numbers, given with normal (Fig. 7) and uniform (Fig. 8) distributions (the data has been normalized).

The calculation of fractal dimension with the suggested method is shown on Fig. 9.

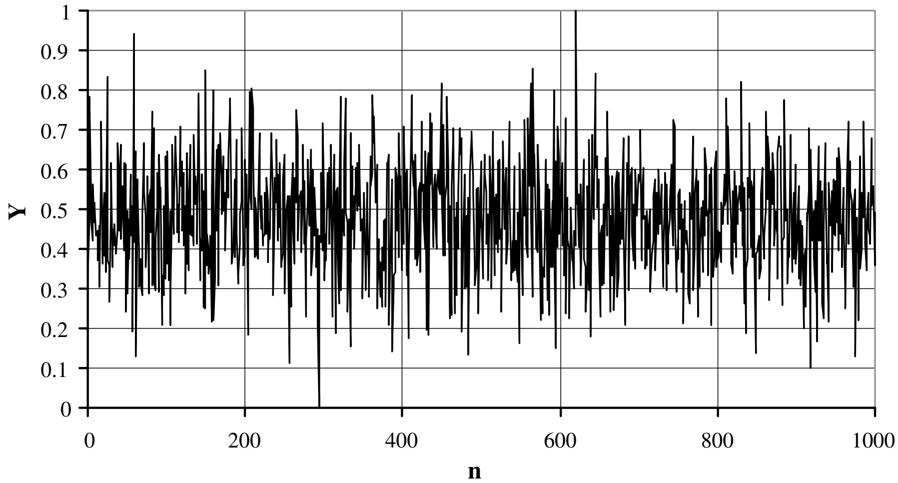

Fig. 7. Realization of random number generator (normal distribution).

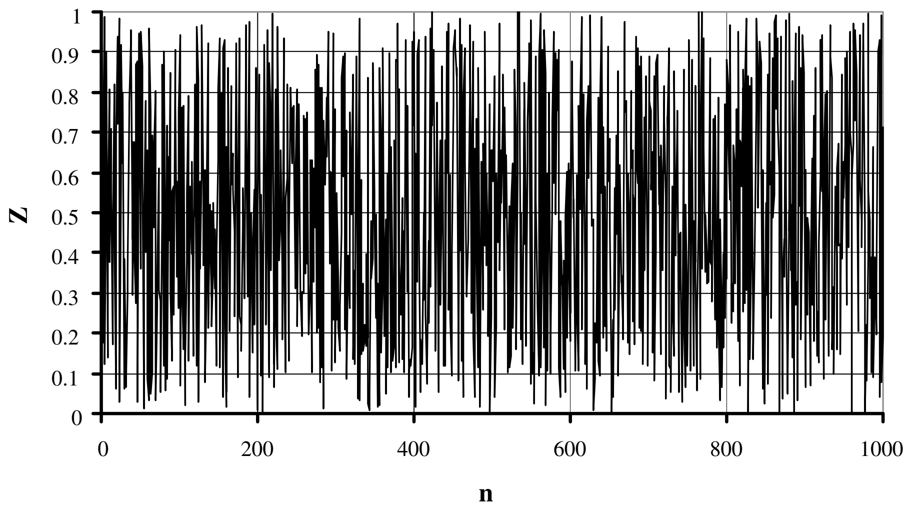

Fig. 8. Realization of random number generator (uniform distribution).

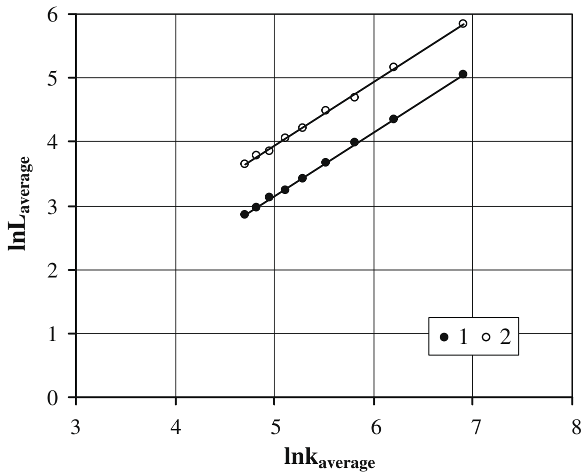

Fig. 9. Dependence $\ln L_{average} = f(\ln k_{average})$ at different distribution: 1 – normal; 2 – uniform.

The initial data straighten well in logarithmic coordinates and the values of fractal dimension, which are defined from slopes of straight lines, are same for normal distribution and for uniform distribution and equal to 2.0. Coefficients A for the given curves differ from each other and are equal to 0.16 and 0.35 correspondingly. The values of specific length l make up 0.16 and 0.35 correspondingly. As it can be seen from the given example, given method allows differentiating processes, which correspond to different functions of distribution.

4. Application of fractal analysis in monitoring of oil wells performance

Oil fields get watered out as development progresses with time. This process is affected by various factors (geophysical properties of a porous media, thermodynamic properties of fluids, offtake rates of gas and oil, number of wells, secondary recovery techniques realization, etc).

Moreover, pressure support is required in order to keep production of oil and gas. In the present times, around 90% of oil is produced from fields with water injection [21]. Water injection supports pressure in the reservoir as well as sweeps the oil from the porous media to the producing wells.

However, water injection can lead to undesirable effects as well, when water fingers out towards the producing well through high permeability streaks leaving oil behind, breaking the flood front and watering out the producer. The flood front stability depends on the difference in viscosities of oil and water, production and injection rates, etc.

At first petroleum and reservoir engineers paid attention to that problem whist injecting water into reservoirs and explained it as pores having heterogeneous spaces. Saffman and Taylor [22] have established by experiments that the flood front stability gets distorted even in non-porous media, when a low viscosity fluid is used to displace a higher viscous fluid. Work [23] shows that stable front can be achieved when the rates of displacement are low.

It is established that creation of fractal structure of fluid displacement can be realized due to fractal structure of the porous media. [24,25]. Fractal dimension, in its place, allows quantitative estimation of the water–oil flood front stability [19].

Technical dilemma of water injection is the fact that huge amount of water needs to be injected in order to pressure support the reservoir, whereas high water volumes lead to flood front instability and early water breakthrough. The mentioned dilemma can be overcome by changing the viscosity of the injected fluid, the viscosity which is close to the viscosity of oil in the reservoir which leads to stable front and maximizes production.

That is why timely forecast of water breakthrough allows management of production in a timely manner.

There are methods of hydrodynamic well testing, which enable estimation of water breakthrough. However, their application often demands high expenditures in which case they become economically non-viable. That is why it is important to analyse well performance using technological parameters such as production rate, pressure, temperature etc.

As for practical example, let's look at application of the suggested fractal factors to retrospective analysis of dynamic of watercut. Watercut is the ration of water produced compared to the volume of total liquids that come out of a producing well (usually an oil well). The content of water in oil that comes out of an oil well has a negative impact on the production of oil and gas. Early time prediction of possible watercut allows to take right decisions to manage the performance of a given well. There are various methods of well testing which help to estimate the movement of water front to a given well, however their realization costs are very expensive and are not always reasonable.

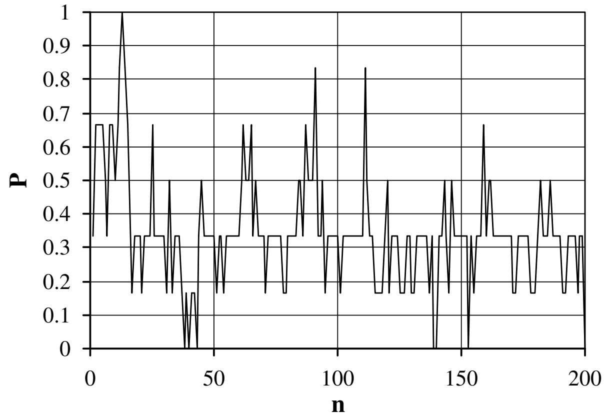

Fig. 10. Dynamics of well head pressure (normalized) in December 2004.

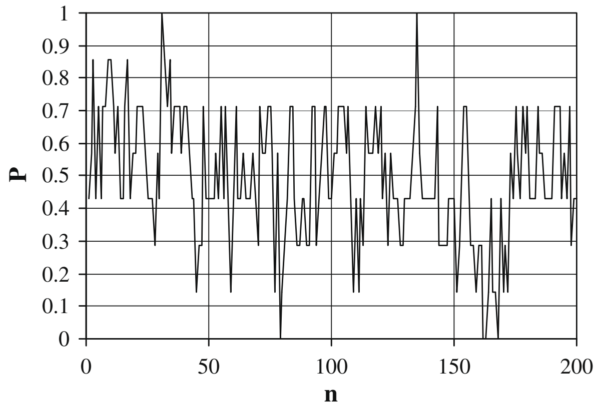

Fig. 11. Dynamics of well head pressure (normalized) in January 2005.

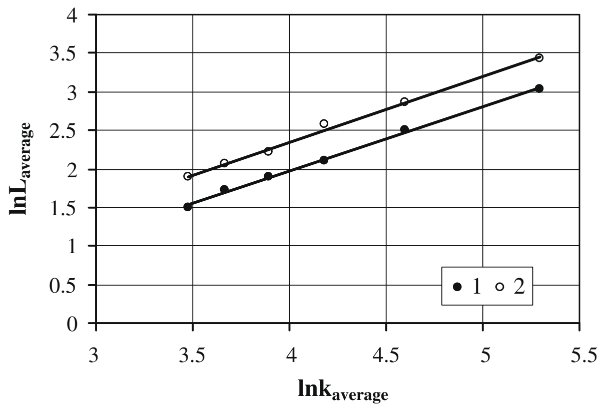

Fig. 12. Change of dependence $\ln L_{average} = f(\ln k_{average})$ as water approaches: 1 – December 2004; 2 – January 2005.

The well data is taken from one of the Azerbaijan oil fields.

Well head pressure measurements are shown on Figs. 10,11 ($n = 200$). The measurements were taken in December 2004 and again in January 2005.

The values of fractal dimension, defined from slopes of straight lines are significantly close: 1.83 and 1.85 (Fig. 12).

Coefficients A for the given curves differ from each other and are equal to 0.26 and 0.35 correspondingly.

The values of the specific length l make up 0.10 and 0.16 correspondingly.

Further well performance (Fig. 13) has shown that change in fluctuation character in the given case was connected to the beginning of water cutting the well. This correlates well with non-equilibrium filtration process of multiphase systems.

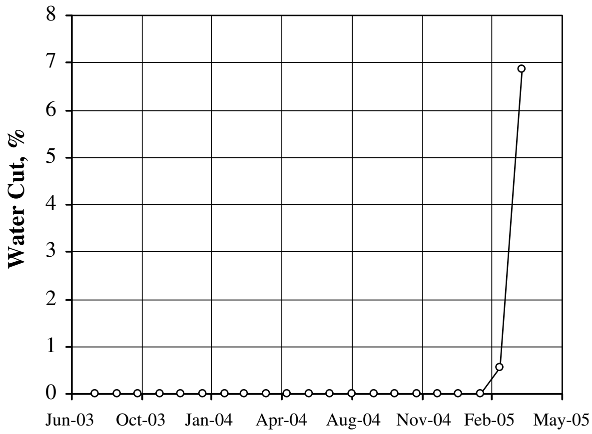

Fig. 13. Dynamics of well watercut.

Hence, on the basis of fractal analysis of technological parameters it is possible to diagnose the changes in well performance.

5. Conclusions

- The suggested method of fractal dimension calculation possesses high accuracy and rapidity of convergence on the limited amount of measurements compared to the method of covering.
- Early diagnosis criteria of qualitative changes in behaviour of different dynamic systems have been suggested, which do not differ by fractal dimension.

Acknowledgement

We express our acknowledgements to Arzu H. Hajizada for helping to prepare this manuscript.

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