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Application of Fractal Analysis of Time Series in Oil and Gas Production

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Abstract: The peculiarities of fractal characteristics' calculations for time series are described in this article. It was suggested an algorithm for calculation of fractal dimension for time series. This method differs from the covering method, considering only the Y coordinate and possessing high accuracy and rapidity of convergence on a limited number of measurements, compared to the method of covering. The criteria of early diagnosis for changes in the condition of hydrodynamic processes that do not vary by fractal dimension have been recommended.

The presented method is applicable for practical engineering calculations with self-affine, chaotic data, usually with a relatively limited number of measurements. It is quite a simple method for calculation of fractal dimensions and the algorithm can be easily realized. The applicability of the proposed algorithm for fractal dimension calculation and early diagnosis criteria of qualitative changes in the behavior of various dynamic systems has been tested both on simulated as well as practical examples of oil and gas production.

Keywords: diagnosis, distribution, fractal dimension, oil and gas wells, pressure fluctuations, time series

1. THE PECULIARITIES OF FRACTAL DIMENSION OF TIME SERIES

The concept of fractals (Mandelbrot, 1967, 1982) in recent times has gained a wide application in numerous spheres of natural science. However, the concept of fractals is not applied solely to topological objects. Time series follow the laws of fractal geometry. The scale invariance for a time series emerges when, during the same period of time T, the process is measured by different steps Δt_n, and new points are added to the curve.

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However, a reliable calculation of fractal characteristics D for a time series, in connection with finiteness of the measurement step, interfaces with the following complications: necessity of long-term studies to acquire a large amount of measurements, and the changing behaviour of a dynamic process during the measurement time (Dubovik et al., 2004). In order to tie together the local changeability of a dynamic process with fractal dimension of a time series, it is necessary to define D locally. It is therefore vital that the method of fractal dimension calculation for time processes possesses rapidity of convergence in the limited number of measurements.

1.1. Fractal Dimension of Time Series

The fractal dimension of time series is traditionally determined in the same way that a curve of some factor is built for continuous curves according to the measurements with a set time step of Δt during the measurement interval of T. The length of the built curve is determined by methods of covering, when the estimated length L, equals δ − the length of the rulers (boxes), multiplied the number of such rulers (boxes) needed to cover the measured curve. The dimension D can be determined using dependence $L \sim \delta^{1-D}$ in logarithmic scales.

However, if the method of covering suits the continuous curves, then this method is very coarse for time dependencies. This is due to the fact that measurements of technological parameters are discrete. The discrete points get connected by a series of lines when building a graph for visualization. The measurements themselves, not the lines connecting them on the graph, should be taken into account when calculating fractal dimensions for a time series (Abbasov et al., 2000).

This study suggests a method for fractal dimension calculation of time series which possesses a rapid convergence on limited amount of measurements as well as simplicity of realization. The suggested algorithm for fractal dimension calculation consists of the following: Let. during a time interval T, the n amount of measurements have been carried out of y_i dynamic process with time step of Δt. Let's put together the sequence of selected measurements out of all measurements according to the following rule:

The first selection contains all measurements of time interval $T : y_1, y_2, y_3, \ldots, y_{n-1}, y_n$;

The second selection consists of measurements, which stand away from each other at the distance of $2 * \Delta t : y_1, y_3, y_5, \ldots, y_{n-2}, y_n$ etc.

The length of time dependencies L for selection we determine as the sum of k absolute values of differences between neighbouring measurements of the investigated parameter for the given selection.

Thus, the length of the first selection will be equal to: $|y_1 - y_2| + |y_2 - y_3| + \ldots + |y_{n-1} - y_n|$.

The length of the second selection will be equal to: $|y_1 - y_3| + |y_3 - y_5| + \ldots + |y_{n-2} - y_n|$, etc.

Hence, the length of a curve is determined dependent on the time step of measurements $\Delta t_m = m * \Delta t$, and not the value of δ. As a consequence of limitation in variation of the measured parameters, the length of the curve L at small values of Δt_m is best described by the following dependence: $L \sim \Delta t_m^{1-D}$. Taking into account that Δt_m is in inverse proportion to $k = \frac{n-1}{m}$ (to the number of lengths which divide the time interval T), we can write down the following: $L \sim k^{D-1}$.

Let's rebuild this dependence in logarithmic coordinates. When the values of k are high, the given dependence lies on a straight line. The slope of this straight line defines the value of D. Experimental studies allow confirmation that the suggested method of fractal dimension calculation possesses high accuracy and rapidity of convergence compared to the methods of covering. Moreover, the given approach allows avoiding the discrepancy units of measured parameters and time when determining the fractal dimension (Mandelbrot, 1997).

The applicability of the suggested algorithm is tested on a modelled example where the values of Weierstrass-Mandelbrot $W(t)$ (Mandelbrot, 1982; and Feder, 1988) fractal dimension D are taken into account as the values of the studied process: $C(t) = \sum_{N=-\infty}^{\infty} \frac{(1-\cos(b^N t))}{b^{(2-D)N}}$, where parameter D varies in the diapason of $1 < D < 2$.

In Abbasov and others (2000), the suggested method was compared with the method of covering. The fractal dimension was calculated according to Weierstrass-Mandelbrot fractal function with given D, both with the suggested as well as with the covering methods. The comparison showed that the covering method needs a significantly larger number of measurements in order the calculated value of D be close to the given one.

1.2. Sensitivity of Calculating Algorithm According to the Number of Data

Cases when $n = 100; 250; 500; 1000; 1500; 2000; 2500$ were looked at when aiming to investigate the influence of a measurement's quantity onto the preciseness of fractal dimension determination. The results of the carried-out calculations are presented in Table 1. It is clear that there is a good correspondence between the set and calculated values of D. At that, the accuracy is higher when the quantity of measurement points increases. When compared to the method of covering, the method suggested in the study possesses high accuracy and rapidity of convergence at limited amount of measurements, together with simplicity.

1.3. Accuracy of Calculating Algorithm

The accuracy of fractal dimension calculation can be enhanced if all existing measurements are taken into account.

Table 1. Comparison of the given and rated values of fractal dimension

No of measurements n	Fractal dimension D			
	D = 1.2	D = 1.4	D = 1.6	D = 1.8
100	1.13	1.30	1.50	1.68
250	1.16	1.36	1.60	1.74
500	1.18	1.38	1.62	1.88
1000	1.17	1.37	1.58	1.77
1500	1.17	1.38	1.59	1.80
2000	1.17	1.38	1.59	1.79
2500	1.18	1.40	1.60	1.80

Thus, for $\Delta t_2 = 2 * \Delta t$ depending on definition of initial point, 2 values for length $L_2 : L_2^1$ and L_2^2 could be calculated, and 2 values for $k_2 : k_2^1$ and k_2^2 correspondingly, where

$$L_2^1 = |y_1 - y_3| + |y_3 - y_5| + |y_7 - y_5| + \dots,$$

$$L_2^2 = |y_2 - y_4| + |y_6 - y_4| + |y_8 - y_6| + \dots.$$

For $\Delta t_3 = 3 * \Delta t$ dependent on selection of an initial point, 3 values $L_3 : L_3^1, L_3^2, L_3^3$ can be calculated, and corresponding to them, 3 values $k_3 : k_3^1, k_3^2, k_3^3$ and so on.

In this case, fractal dimension D can be identified by arithmetical mean values L_m and k_m from the following dependence: $L_{average} \sim k_{average}^{D-1}$,

Comparing Figures 1 and 2 (function $C(t)$ at $D = 1.4$, $b - 1.5$ and $n = 250$), it is seen that the dependence $\log L - \log k$ is significantly better

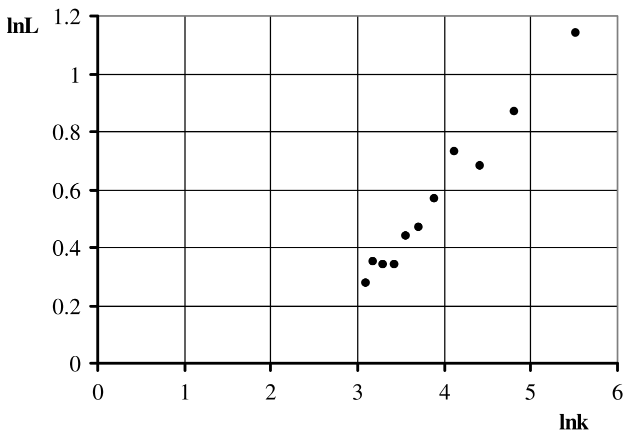

Figure 1. Dependence $\ln L = f(\ln k)$ for $n = 250$, $b = 1.5$, at $D = 1.4$.

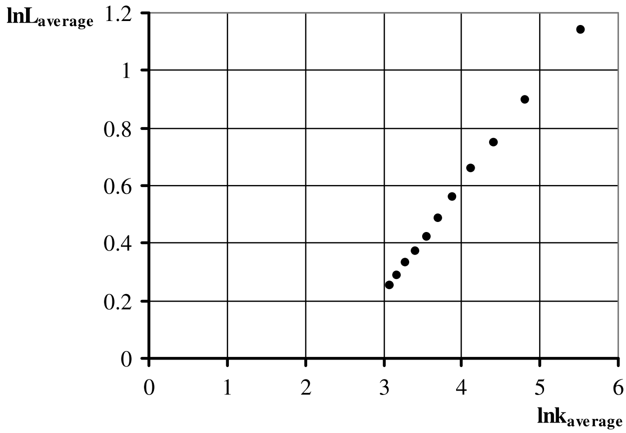

Figure 2. Dependence ln $L_{average} = f(\ln k_{average})$ for $n = 250$, $b = 1.5$, at $D = 1.4$.

straightened when the length of the L and k curves is averaged (when all the existing measurements are taken into account).

2. ANALYSIS OF PROCESSES WITH CLOSE FRACTAL DIMENSIONS

Fractal dimension and Hurst exponent are used as integrated factors when solving practical tasks. They characterize the peculiarities of the investigated object. This allows diagnosing the changes in conditions of technological processes.

The deficiency of the fractal analysis, in its classical statement, is the fact that numerous technological processes, which are characterized by factors' fluctuations (in particular a wide range of stationary and chaotic processes) do not divide on fractal coordinate space (Klikushin, 2000).

In order to increase the effectiveness of process condition diagnosis, a method, shown below, is suggested.

In cases when time series $y(t)$ and $z(t)$ are compared, the difference between their fractal dimensions is small. Their distinction can be estimated according to changes in coefficient A from the following dependence: $L = Ak^{D-1}$.

At that, initial time series $y(t)$ and $z(t)$ are normalized, each of them according to their maximum and minimum values:

$$Y_i = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}} \text{ and } Z_i = \frac{z_i - z_{\min}}{z_{\max} - z_{\min}}.$$

Such transformation (normalization) of initial time series is not reflected in the value of their fractal dimensions, and, at the same time, allows one to track the change in local fractal dimensions of the same process at different stages of development, and to compare different characteristics of the process, which were measured at the same period of time.

Sensitivity of coefficient A to non-significant changes of the investigated process, which are difficult to detect by the value of fractal dimension, can be seen from the following example:

Lets regard the normalized values of function $C(t)$ at the same values $D = 1.4$, $n = 1000$ and varying b : 1.5 and 1.2. The initial data are well straightened in logarithmic coordinates and the values of fractal dimension, which are defined from the slope of straight line, do not differ: 1.37 and 1.37. At the same time, coefficients A for the given curves, after expansion from logarithm, differ significantly, equalling 0.398 and 0.284, correspondingly.

3. APPLICATION OF FRACTAL ANALYSIS IN MONITORING OF OIL WELLS PERFORMANCE

The abilities of fractal geometry found their application in studies of different processes of oil and gas production (Hardy and Beier, 1994; Mirzajanzadeh and Sultanov, 1995; Mirzajanzadeh and Shahverdiyev, 1997; Mirzajanzadeh et al., 2004).

Fractal analysis is used to interpret the results of geophysical and hydrodynamic measurements, such as seismic measurements, logging, fluid samples, coring, and pressure transient tests (Katz and Thompson, 1985).

As a practical example, let's look at application of the suggested fractal factors to retrospective analysis of dynamic of watercut. Oil fields get watered-out as development progresses with time. Watercut is the ratio of water produced compared to the volume of total liquids that come out of a producing well (usually an oil well). This process is affected by various factors (geophysical properties of a porous media, thermodynamic properties of fluids, offtake rates of gas and oil, number of wells, secondary recovery techniques realization, etc.). The content of water in oil that comes out of an oil well has a negative impact on the production of oil and gas. Early time prediction of possible watercut allows to take right decisions to manage the performance of a given well. There are various methods of well testing that help to estimate the movement of water front to a given well; however, their realization costs are very expensive and are not always reasonable.

The well data are taken from one of the Azerbaijan oil fields. Well head pressure measurements are shown on Figure 3 ($n = 200$). The measurements were taken in December, 2004, and again in January, 2005. The values of the fractal dimension, defined from slopes of straight lines, are significantly close: 1.83 and 1.85. Coefficients A for the given curves differ from each other and are equal to 0.26 and 0.35, correspondingly.

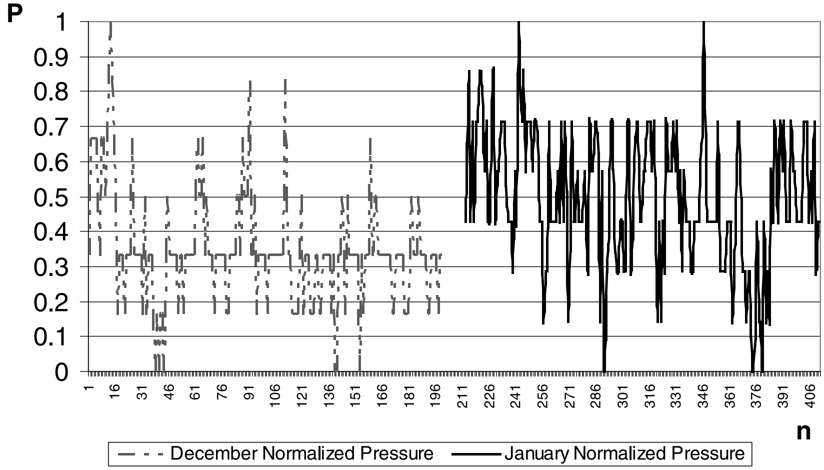

Figure 3. Dynamics of well head pressure (normalized).

Further well performance (Figure 4) has shown that a change in fluctuation character in the given case was connected to the beginning of watercutting the well in February, 2005. This correlates well with the non-equilibrium filtration process of multiphase systems.

Hence, on the basis of fractal analysis of technological parameters, it is possible to diagnose the changes in well performance.

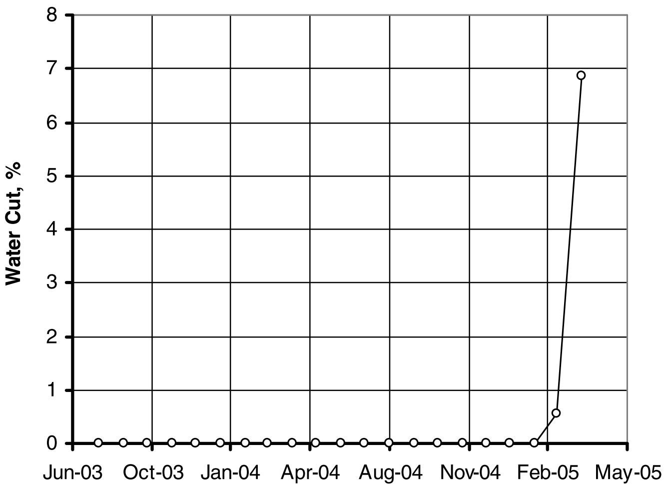

Figure 4. Dynamics of well watercut.

CONCLUSIONS

- The suggested method of fractal dimension calculation possesses high accuracy and rapidity of convergence on the limited amount of measurements compared to the method of covering.
- Early diagnosis criteria of qualitative changes in behavior of different dynamic systems have been suggested, and they do not differ by fractal dimension.

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